

MULTIRESOLUTION ANALYSIS OF INCOMPRESSIBLE FLOWS INTERACTION WITH FORCED DEFORMABLE BODIES

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Abstract. In the present investigation, a space adaptive multiresolution method is developed to solve the incompressible two-dimensional Navier-Stokes equations in the vorticity-stream-function formulations including the penalization term. The new method is based on a multiresolution analysis which allows to reduce the number of active grid points significantly by refining the grid automatically via nonlinear thresholding of the wavelet coefficients in a one-to-one correspondence with the grid points. To study the accuracy of the method, dipole collision with a straight wall is considered as a benchmark, a good agreement between the results of adaptive simulations and that of uniform grid solver is obtained. The grid adaptation strategy uses an estimation of the local regularity of the solution via wavelet coefficients at a given time step. An extension to interactions with forced deformable bodies, i.e., swimming of a fish, is done using the volume penalization method. A Lagrangian structure grid with prescribed motion cover the deformable body interacting with surrounding fluid due to hydrodynamic forces and moment calculated on an Eulerian reference Cartesian grid. The results of swimming fish are compared with those of Gazzola et al. where a uniform grid is used. The obtained results show that the CPU-time of the adaptive simulations can be significantly reduced with respect to simulations on a regular grid. Nevertheless the accuracy order of the underlying numerical scheme is preserved.

1 INTRODUCTION

The aim of present investigation is to develop a reliable self-adaptive numerical method for direct simulation of incompressible flows. Conventional methods for spatial discretiza-

tion of the PDEs (e.g., finite differences, finite volumes and finite elements) have limited order of accuracy especially near boundaries, but they are more flexible in dealing with complex geometries over a suitable grid. On the other hand standard spectral methods which are widely used in direct numerical simulation of turbulence are limited to Cartesian grids. One can recognize the poor spectral localization (good spatial localization/resolution) of the former methods while good spectral localization (poor spatial localization/resolution) of the latter methods [4]. The limitation of mentioned methods, for problems with widely disparate spatial scales, has encouraged the researcher to use alternative methods, with limited accuracy but good spatial localization in regions where high gradient of flow variables is present. Adaptive methods can be divide into r-type (a fixed number of grid points are redistributed), h-type (regridding is performed occasionally) and p-type (the degree of the polynomial representing the solution is locally increased) each one with their own advantages and disadvantages as detailed in literatures. Among different methods for grid adaptation h-type refinement proved to be more advantageous in terms of error control. Among different error-estimating adaptation strategies (which most of them belong to the finite element family) wavelet-based numerical methods have proved to be an efficient tool in developing adaptive numerical methods which control the global (usually L_2) approximation error. Wavelet transforms allow to estimate the local regularity of the solutions to a given PDE, with a very efficient algorithm, and thus can define auto-adaptive discretization with local mesh refinement [9]. Liandrat and Tchamitchian [1] proposed the first wavelet-based adaptive method for numerical simulation of PDEs. The currently existing wavelet-based algorithms can be classified as pure wavelet methods and wavelet optimized grid methods. Pure wavelet methods, employ wavelets directly for discretization of the governing equations. On the other hand, wavelet optimized grid (WOG) methods [11] combine classical discretizations of considered equations (e.g., finite differences or finite volumes) with wavelets, which are used to define the adaptive grid. See [6] and [13] where a finite volume discretization of governing equations combined with cell-averaged interpolating wavelet transform for grid adaptation. In the present work the method of adaptive multiresolution analysis will be applied to the Navier-Stokes equations in vorticity and stream-function formulation. However the concepts are also applicable to the primitive variables. Thus similar to WOG methods the role of the wavelet transform is the adaptation of the grid and the fast interpolation of flow variables at new unknown points. A second-order central finite difference method with symmetric stencil over an adaptive Cartesian grid is used for spatial discretization of the equations. Finite difference method represents a suitable combination with the multiresolution analysis based on the Harten's point-value wavelet transform. The concept of symmetric stencils will lead to intermediate (hung) points, that their values can be interpolated accurately via inverse wavelet transform, see [5]. After validation of the developed adaptive multiresolution solver, using the results of previous studies of dipole-wall collision, an extension to fish swimming via the volume penalization method will be presented. Volume penalization method is a sub branch of immersed

boundary methods, see [7] for a complete review of these methods. As a starting point in the present work we take the two-dimensional vorticity stream-function solver developed in [12] for a uniform grid and the adaptive solver developed in [15] for simulation of the flow inside curved geometries. The code is developed in FORTRAN and is accessible for all [19]. The manuscript is organized as follows; In the following a summary of governing equations, multiresolution analysis, discrete wavelet transforms and the idea of point selection by filtering of the wavelet coefficients will be presented. After that for validation of the solver the results of the dipole collision with straight wall is compared with previous studies. Next a test case from fish swimming will demonstrated as application. Finally, conclusions and perspectives will be discussed.

2 GOVERNING EQUATIONS

The governing equations of the incompressible flows are the Navier-Stokes equations. In two-dimensional problems the vorticity ω and stream-function ψ formulation is more efficient than primitive variables. By taking the curl of the Navier-Stokes equations, one obtains the vorticity transport equation:

$$\partial_t \omega + (\mathbf{u} \cdot \nabla) \omega = \nu \nabla^2 \omega + \nabla \times \mathbf{F}, \quad \mathbf{x} \in \Omega \in \mathbb{R}^2 \quad (1)$$

where $\omega(\mathbf{x}, t) = \nabla \times \mathbf{u} = v_x - u_y$ denotes the vorticity, Ω is the spatial domain of interest, $\mathbf{u}(\mathbf{x}, t)$ is the velocity field, $\nu = \mu/\rho_f > 0$ is the kinematic viscosity of the fluid, ρ_f is the density and $\mathbf{F}(\mathbf{x}, t)$ is a source term. For a complete description of a particular problem, the above equations need to be complemented to describe an initial/boundary value problem (IBVP). The equation is parabolic in time and the velocity components are $(u, v) = (\partial_y \psi, -\partial_x \psi)$ with ψ being the stream-function, satisfying a Poisson equation

$$-\nabla^2 \psi = \omega \quad (2)$$

which is an elliptic equation in space. The penalization term for unit mass of the fluid reads,

$$\mathbf{F} = -\eta^{-1} \chi(\mathbf{u} - \mathbf{u}_P) \quad (3)$$

where $\mathbf{u}_P(\mathbf{x}, t)$ is the velocity field of the immersed body. The Navier-Stokes equations are written for unit mass of the fluid, therefore the dimension of the terms like \mathbf{F} is acceleration, i.e., $[LT^{-2}]$. Penalization parameter η is the porosity (permeability) coefficient of the immersed body with dimension $[T]$. The mask (characteristic) function χ is dimensionless and describes the geometry of the immersed body.

$$\chi(\mathbf{x}, t) = \begin{cases} 1 & \mathbf{x} \in \Omega_b \\ 0 & \mathbf{x} \in \Omega_f \end{cases} \quad (4)$$

where Ω_f represents the domain of the flow and Ω_b represents the immersed body in the domain of the solution. The solution domain $\Omega = \Omega_f \cup \Omega_b$ is governed by the Navier-Stokes equations in the fluid regions and by Darcy's law in the penalized regions, when $\eta \rightarrow 0$.

3 MULTIREOLUTION ANALYSIS

Denoting by $E(\Delta t)$ the discrete time evolution operator, the global algorithm can schematically be summarized by

$$\omega^{n+1} = E(\Delta t) \left[M^{-1} \cdot S \cdot T(\epsilon) \cdot M \right] \omega^n \quad (5)$$

where M and M^{-1} are the direct (WT) and inverse (IWT) wavelet transform operators. $T(\epsilon)$ is the thresholding operator and S represents the safety zone operator. For an Euler explicit time integration we have

$$E(\Delta t)\omega^n = \omega^n + \Delta t \, RHS(\omega^n). \quad (6)$$

where RHS operator contains all the terms of the considered evolutionary equation to be integrated except time derivative. The summary of the multiresolution method is given in Algorithm 1. Some important notes are given in the following: **(1)** Before interpolation of the values of an independent variables via IWT (from the coarsest level up to the finest level) in some grid points (with wavelet coefficients equal to zero, $d = 0$), it is necessary to mark all the intermediate necessary points for having a consistent WT, (from the finest level down to the coarsest level) and adding them to the list of the points to be interpolated. **(2)** In time integration via multi-step methods such as Runge-Kutta family before calculation of spatial derivatives at intermediate steps, the value of u^* for the hung points, must be interpolated again from the new values of active points. But 6-(a) and 6-(b) will be done once in each time step. **(3)** In the case of the two-dimensional Navier-Stokes equations in vorticity stream-function formulation, before calculation of the spatial derivatives it is necessary to solve an elliptic equation, i.e., Eq. (2) for updating stream-function, for more details see [17].

3.1 Biorthogonal wavelet transform

To explain the concept of WT, we consider the case of Harten's point values representation [3] over a uniform grids, which is well adapted for finite difference methods, versus Harten's cell average method which is more suitable for finite volume methods. By considering in a unit interval, the hierarchy of uniform dyadic grids will obtain from

$$X_j = \{x_{j,i} \in \mathbb{R} : x_{j,i} = i2^{-j}, i = 0, \dots, 2^j\}, \quad j = 0, \dots, J \quad (7)$$

with spacing 2^{-j} , where j is the level and i represents the position. The number of points must always be odd ($N = 2^J + 1$) to have a point in the middle. A given discrete function $f(x)$ can be represented with the use of wavelet basis as follows

$$f(x) = \sum_{i=0}^{2^J} f_{0,i} \Phi_{0,i}(x) + \sum_{j=0}^J \sum_{i=0}^{2^j} d_{j,i} \Psi_{j,i}(x) \quad (8)$$

Algorithm 1 Multiresolution analysis

1. Start from an initial condition over a dyadic grid
 2. Apply WT to the active points (from the finest level down to the coarsest level) to compute the wavelet coefficients of the independent variable
 3. Perform thresholding $T(\epsilon)$ to remove all the points from the list of the active points which their wavelet coefficients are below the corresponding threshold ϵ_j
 4. Add safety zone to the list of the new active points
 - (a) Add neighbor points at the same and one above levels
 - (b) Guarantee the gradedness of the new active points (optional)
 - (c) Add necessary points to the current list of the active points, for having a consistent direct or inverse WT
 5. Apply IWT to the new active points to compute the values of the independent variables (or interpolate the values of all newly added points via IWT with $d = 0$)
 6. Perform the time evolution of the independent variable for all the active points
 - (a) Search for the nearest active point to determine $dist$ for all active points
 - (b) Check for the existence of all other neighbors of the active points with distance $dist$, mark all the missing points as the hung points
 - (c) Interpolate the values of the hung points via IWT with $d = 0$
 - (d) compute the spatial derivatives for the given PDE via FDM with symmetric stencils
 7. Go to step 2, if $T < T_{end}$
-

where the orthonormal basis are considered scaling functions $\Phi_{j,i}$ and wavelets $\Psi_{j,i}$. Interpolating wavelet coefficients are defined as

$$d_{j,i} = \langle f, \Psi_{j,i} \rangle = f_{j+1,2i+1} - \tilde{f}_{j+1,2i+1} \quad (9)$$

where cubic (third-order) interpolation can be used as follows,

$$\tilde{f}_{j+1,2i+1} = \frac{-f_{j,i-1} + 9f_{j,i} + 9f_{j,i+1} - f_{j,i+2}}{16} \quad (10)$$

near boundaries forward/backward stencil must be used for interpolation, for more details see [17].

3.2 Filtering of wavelet coefficients

Given a threshold parameter for the finest ϵ_J or the coarsest ϵ_1 level, data compression will be obtained by thresholding of the detail coefficients, also called nonlinear filtering, applied to wavelet coefficients in wavelet space. After performing the direct transform, wavelet coefficients smaller than a threshold are set to zero and the corresponding point can be eliminated from the set of the points, in other words we can find the value of that point by interpolation and the error remains bounded by the threshold value.

$$d_{j,i} = \begin{cases} 0 & \text{if } |d_{j,i}| \leq \epsilon_j, \\ d_{j,i} & \text{else} \end{cases} \quad (11)$$

where $\epsilon_j = \epsilon_J 2^{D(j-J)} = \epsilon_0 2^{D(j)}$, $D = 1, 2, 3$ is the dimension of the problem, and J denotes the maximum level. After nonlinear filtering in wavelet space the given function $f(x)$, can be reconstructed $\bar{f}(x)$, just with the significant wavelet coefficients corresponding to the points where the function is not regular. Those points must be kept to guaranty the boundedness of the error introduced due to filtering and eliminating non necessary points. Following Donoho [2], it can be shown that for a sufficiently smooth function $f(x)$, the error is bounded by threshold, i.e., $|f(x) - \bar{f}(x)| \leq c_1 \epsilon_0$. Consider a non-periodic one-dimensional function $f(x)$ over $[0, 1]$

$$f(x) = \begin{cases} 8.1e^{1/4}e^{-|x-1/2|} & 0.0 \leq x < 0.25 \\ 9e^{-|x-1/2|} & 0.25 \leq x < 0.75 \\ e^{-|x-1/2|}(16x^2 - 24x + 18) & 0.75 \leq x \leq 1.0 \end{cases} \quad (12)$$

with a jump at $x = 0.25$, a jump in the first derivative at $x = 0.5$ and a jump in the second derivative at $x = 0.75$. Consider also a Gaussian function, $f(x) = \exp((x - 0.5)/\delta)^2$ where $x \in [0, 1]$. Their sparse point representations, with the use of cubic interpolating wavelet ($P_{WT} = 4$) transform, for $J = 10$, filtered with threshold $\epsilon = 1 \times 10^{-3}$ are illustrated in Fig. 1 (a) and (b). A good compression and error bounded by threshold can be seen, for more details see [17].

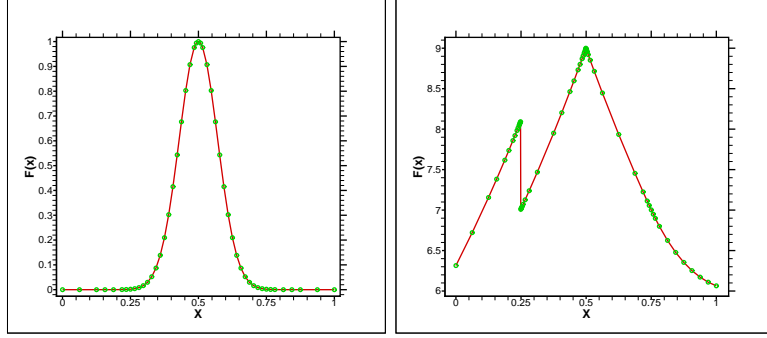


Figure 1: Sparse point representation of 1D functions, obtained by WT with cubic interpolation ($J = 10$), filtered with threshold $\epsilon = 1 \times 10^{-3}$. The green dots (marked ●) show the retained grid points. (Left) Gaussian function, compression = 95%, L_∞ -Error $\leq 1 \times 10^{-4}$. (Right) Function (12), Compression = 94%, L_∞ -Error $\leq 5 \times 10^{-5}$.

4 VALIDATION

In the present investigation the problem of dipole-wall collision studied by Clercx et al. [10] is chosen as a benchmark computation for validation of the proposed algorithm. The simulation is performed in a square domain $[0, 2] \times [0, 2]$ with four rigid walls at $(x, y = 0 \text{ \& } x, y = 2)$. The flow is initialized in the form of two shielded Gaussian mono-polar vortices, where their centers placed at a distance 0.2 apart. The vorticity distribution in each monopole is given by

$$\omega(0, \mathbf{x}_n) = \omega_e(1 - r^2/r_0^2) \exp(-r^2/r_0^2) \quad (13)$$

where r_0 is the core radius, $r = \|\mathbf{x} - \mathbf{x}_n\|$ with \mathbf{x}_n being the position of the vortex center. The two isolated monopoles are located at $\mathbf{x}_1 = (1, 1.1)$ and $\mathbf{x}_2 = (1, 0.9)$, demanding that the root mean square (rms) velocity is initially equal to unity ($E = 2$) yields the amplitude of each isolated monopole, $\omega_e = \pm 299.528385375226$ [14]. The core radius of the shielded monopoles is set to $r_0 = 0.1$. The integral-scale Reynolds number for the initial field is given by $Re = U_{rms}L/\nu$ where the characteristic length scale is set to the half-height of the domain, $L = 1$ and the characteristic velocity to the initial root mean square velocity, $U_{rms} = 1$. The time evolution of the dipole is calculated by the developed multiresolution finite difference solver with threshold $\epsilon = 10^{-3}$ and maximum grid level $J = 11$ for Reynolds 1000. The evolution of the vorticity isolines and the corresponding adaptive grid starting from the initial condition at $t = 0$ up to $t = 1$, is shown in Fig. 2. Comparisons of the total energy $E(t) = \frac{1}{2} \int_\Omega |\mathbf{u}(\mathbf{x}, t)|^2 d\mathbf{x}$ and the total enstrophy $Z(t) = \frac{1}{2} \int_\Omega |\omega(\mathbf{x}, t)|^2 d\mathbf{x}$ between the uniform grid solver and the multiresolution computation with thresholds, $\epsilon = 10^{-3}$ and $\epsilon = 10^{-4}$, with maximum grid level $J = 9$ are plotted in Fig. 3 (a) and (b), respectively. The agreement between the uniform grid solver and the multiresolution solver is perfect and the results for $\epsilon = 10^{-3}$ and $\epsilon = 10^{-4}$ are almost identical. Therefore we will use $\epsilon = 10^{-3}$ for all multiresolution computations. A convergence study for the total enstrophy $Z(t)$ (with the uniform grid solver) for Reynolds

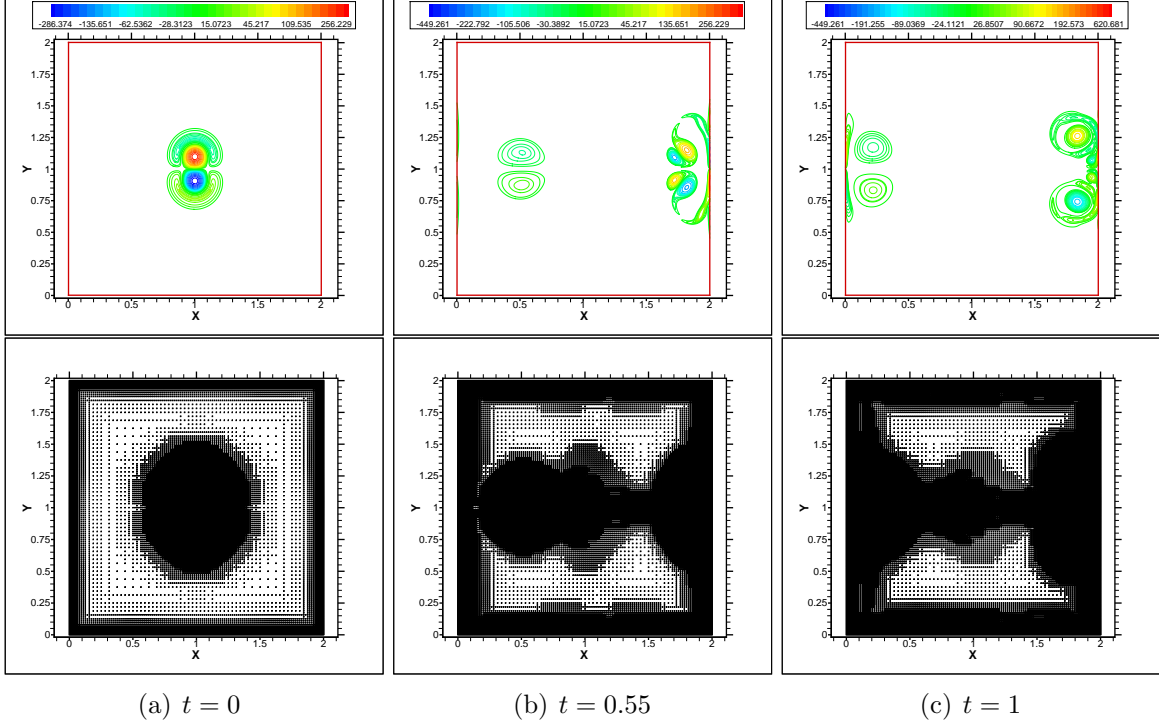


Figure 2: The evolution and collision of the vortices (represented by the colored isolines) with walls (up) and the corresponding adaptive grid (down), maximum grid level $J = 11$ in each direction, threshold $\epsilon = 10^{-3}$, for Reynolds 1000.

1000, with different grid spacings, i.e., maximum level in each direction $J = 8, 9, 10, 11$, is performed. The simulation with pseudo-spectral solver of Clercx is taken as the reference solution [10]. The results of the present computation are illustrated in Fig. 3 (c). It can be observed that by increasing the number of grid points the curves become closer and closer, we hope the results of $J = 12$ will match with that of Clercx et al. [10].

5 APPLICATION

Anguilliform swimming presented in Gazzola et al. [16] is considered as application for the proposed algorithm. The details of our fluid/solid interaction algorithm is given in [18]. A periodic swimming law is defined by fitting the backbone of the fish to a given curve $y(x, t)$ keeping the backbone length l_{fish} fixed. Let ξ be the arclength over curvilinear coordinate of the deformed backbone ($0 \leq \xi \leq l_{\text{fish}}$). For points uniformly distributed $\Delta\xi = l_{\text{fish}}/(N - 1)$ over the backbone, y is given by

$$y(x, t) = a(x) \sin(2\pi(x/\lambda + ft)) \quad (14)$$

where λ is the wavelength, f is the frequency of the backbone and $a(x)$ is the envelope $a(x) = a_0 + a_1x + a_2x^2$ where x is defined by inverting the arclength integral, i.e., $\Delta x = \Delta\xi/\sqrt{1 + (\partial y/\partial x)^2}$. The geometry of the fish is presented in [18]. The parameters used

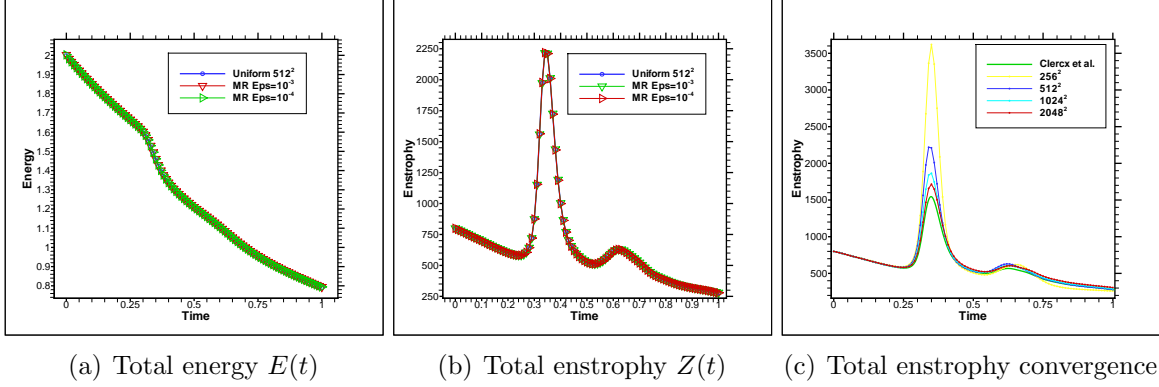


Figure 3: Comparisons of the total energy (a) and the total enstrophy (b) between the uniform grid solver and the multiresolution computation with thresholds, $\epsilon = 10^{-3}$ and $\epsilon = 10^{-4}$, for Reynolds 1000 and maximum grid level $J = 9$ in each direction for all simulations. Convergence study (c) for the total enstrophy $Z(t)$ toward the data from Clercx et al. [10] with the present finite difference computations (Uniform/MR solver) for Reynolds 1000 and maximum grid level $J = 8, 9, 10, 11$ in each direction.

by Gazzola et al. [16] for the kinematics of the fish are as follows; $\lambda = 1$, $f = 1$, $a_2 = 0$, $a_1 = 0.125/(1 + c)$, $a_0 = 0.125c/(1 + c)$ and $c = 0.03125$. The buoyancy is equal to zero, i.e., $\rho_b = \rho_f$. The viscosity of the fluid is set to $\nu = 1.4 \times 10^{-4}$ resulting in a Reynolds number approximately $Re \approx 3800$, with an asymptotic mean velocity $U_{\text{forward}} \approx 0.52$. The simulations of Gazzola et al. [16] are carried out on a rectangular domain $(x, y) \in [0, 8l_{\text{fish}}] \times [0, 4l_{\text{fish}}]$ with resolution of 4096×2048 and penalization parameter $\eta = 10^{-4}$. We are performing our simulations on a rectangular domain $(x, y) \in [0, 8l_{\text{fish}}] \times [0, 8l_{\text{fish}}]$ by imposing penalization parameter inside the body equal to $\eta = 10^{-3}$ with maximum resolution of 1025×1025 and $\Delta t = 5 \times 10^{-4}$. The centroid of the fish is initially positioned at $x_{\text{cg}} = 0.9L_x$ and $y_{\text{cg}} = 0.5L_y$ in our simulations. We impose two degree of liberty fixing the angular velocity equal to zero. The simulations start with the body and fluid at rest. The forward velocities of the center of the mass computed with different methods/parameters are compared in Fig. 4 (left). The evolution of the number of active, significant (corresponding to the retained points after filtering of wavelet coefficients), safety zone, hung and interpolated points for the wavelet transform during the computation with the multiresolution solver is demonstrated in Fig. 4 (right). The number of the points used in multiresolution analysis over the uniform simulation resulting in a compression more than 95%. Fig. 5 shows different views of the adaptive grids colored by vorticity and the mask χ function at $t = 6$.

6 CONCLUSIONS

In the present investigation, a space adaptive multiresolution method is developed to deal with two-dimensional unsteady incompressible flows. The new method is based on a multiresolution analysis which allows to reduce the number of active grid points significantly by refining the grid automatically via nonlinear thresholding of the wavelet

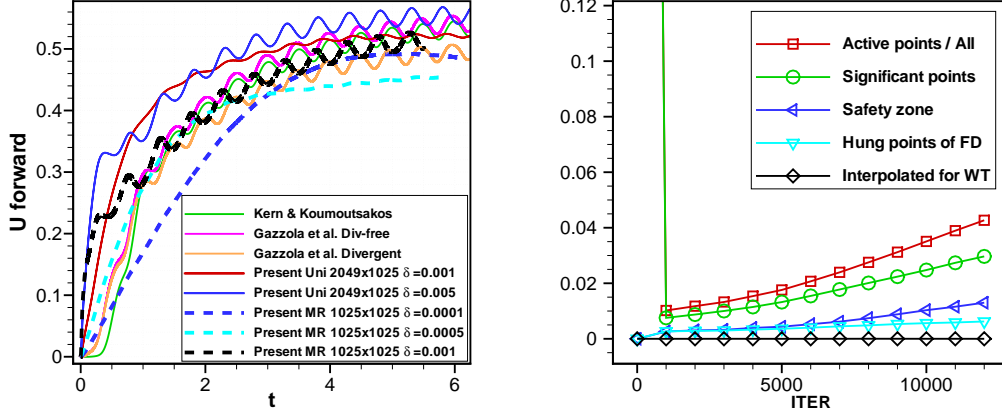


Figure 4: (left) Anguilliform 2D swimmer's ($\lambda = f = 1$) forward velocity U . Solid lines indicate reference simulations performed by Kern and Koumoutsakos (green) [8], Gazzola et al. (pink and brown) [16] and Ghaffari et al. (red and blue) [18]. Dashed lines represent the results with the proposed algorithm. (right) The evolution of the number of active, significant (corresponding to the retained points after filtering of wavelet coefficients), safety zone, hung and interpolated points for the wavelet transform during the computation with the multiresolution solver, with maximum grid level $J = 10$ in each direction (1025^2 grid points).

coefficients in a one-to-one correspondence with the grid points. In the present work the concept of adaptive multiresolution method is applied to the vorticity and stream-function formulation. A second-order central finite difference method with symmetric stencil over an adaptive Cartesian grid is used for spatial discretization of the equations. After validation of the proposed algorithm an extension to deal with fluid interaction with forced deformable bodies, i.e., swimming of a fish, is done using the volume penalization method. A Lagrangian structure grid with prescribed motion cover the deformable body interacting with surrounding fluid due to hydrodynamic forces and moment calculated on an Eulerian reference Cartesian grid. The results of swimming fish are compared with those of Gazzola et al. where a uniform grid is used. The obtained results show that the CPU-time of the adaptive simulations can be significantly reduced with respect to simulations on a regular grid. Nevertheless the accuracy order of the underlying numerical scheme is preserved. Creation of a data-structure for memory deallocation is proposed as a perspective for researchers, see [13]. The code is developed in FORTRAN and is accessible for all [19].

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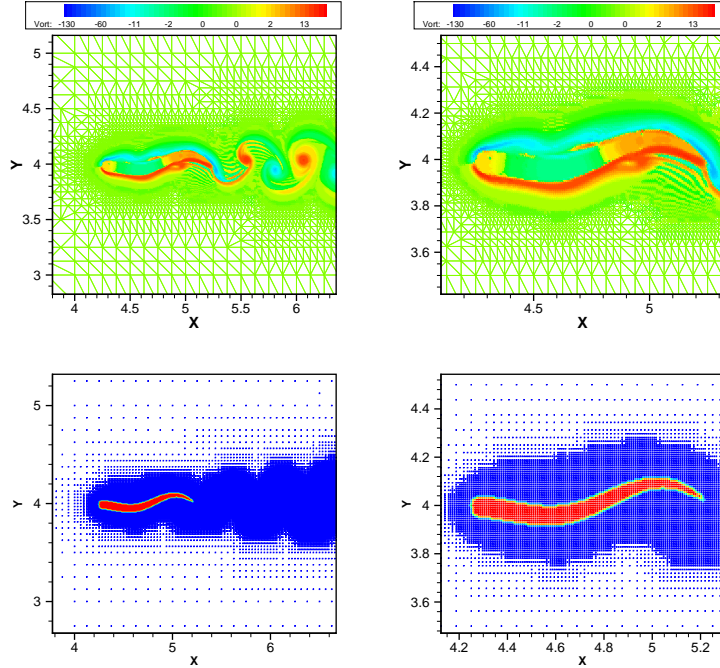


Figure 5: The adaptive grids colored by vorticity (up) and colored by mask χ (down) at $t = 6$ (zoom in from left to right) where $(x, y) \in [0, 8l_{\text{fish}}] \times [0, 8l_{\text{fish}}]$ by imposing penalization parameter inside the body equal to $\eta = 10^{-3}$, with maximum grid level of $J = 10$ in each direction, viscosity $\nu = 1.4 \times 10^{-4}$.

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